

AP Calculus BC

Integration by Parts 2

$$1) \int x^2 \cos x \, dx = h(x) - \int 2x \sin x \, dx$$

$$\begin{array}{ll} u = x^2 & v = \sin x \\ du = 2x \, dx & dv = \cos x \, dx \end{array}$$

$$h(x) = x^2 \sin x$$

$$2) \int x \sin(5x) \, dx$$

$$\begin{array}{c} \frac{D}{x} \quad \frac{I}{\sin(5x)} \\ -1 \quad \downarrow \\ 0 \quad \downarrow \end{array}$$

$$\begin{array}{l} \sin(5x) \\ -\frac{1}{5} \cos(5x) \\ -\frac{1}{25} \sin(5x) \end{array}$$

$$\boxed{-\frac{1}{5}x \cos(5x) + \frac{1}{25} \sin(5x) + C}$$

$$3) \int x \csc^2 x \, dx$$

$$\begin{array}{c} \frac{D}{x} \quad \frac{I}{\csc^2 x} \\ -1 \quad \downarrow \\ 0 \quad \downarrow \end{array}$$

$$\begin{array}{l} -\cot x \\ -\ln|\sin x| \end{array}$$

$$\boxed{-x \cot x + \ln|\sin x| + C}$$

$$4) \frac{dy}{dx} = x \sec^2 x \quad y=1, x=0$$

$$\begin{array}{c} \frac{D}{x} \quad \frac{I}{\sec^2 x} \\ -1 \quad \downarrow \\ 0 \quad \downarrow \end{array}$$

$$\begin{array}{l} \tan x \\ -\ln|\cos x| \end{array}$$

$$\begin{aligned} y &= x \tan x + \ln|\cos x| + C \\ 1 &= 0 + \ln|\cos 0| + C \\ 1 &= C \end{aligned}$$

$$\boxed{y = x \tan x + \ln|\cos x| + 1}$$

$$5) \int_0^{\pi} t \sin 3t \, dt$$

$$\begin{array}{c} \frac{D}{t} \quad \frac{I}{\sin 3t} \\ -1 \quad \downarrow \\ 0 \quad \downarrow \end{array}$$

$$\begin{array}{l} \sin 3t \\ -\frac{1}{3} \cos 3t \\ -\frac{1}{9} \sin 3t \end{array}$$

$$\left[-\frac{1}{3}t \cos 3t + \frac{1}{9} \sin 3t \right]_0^{\pi}$$

$$\left[-\frac{1}{3}(\pi)(-1) + 0 \right] - \left[0 + 0 \right]$$

$$\boxed{\frac{\pi}{3}}$$

$$6) \int_0^1 (x^2+1) e^{-x} \, dx$$

$$\begin{array}{c} \frac{D}{x^2+1} \quad \frac{I}{e^{-x}} \\ -2x \quad \downarrow \\ +2 \quad \downarrow \\ 0 \quad \downarrow \end{array}$$

$$\begin{array}{l} -e^{-x}(x^2+1) \\ -e^{-x}(2x) \\ -e^{-x}(2) \end{array}$$

$$\left[-e^{-x}(x^2+1) - e^{-x}(2x) - e^{-x}(2) \right]_0^1$$

$$\left[-\frac{2}{e} - \frac{2}{e} - \frac{2}{e} \right] - \left[-1 - 2 \right]$$

$$\boxed{-\frac{6}{e} + 3}$$

$$7) \int_1^e \frac{\ln x}{x^2} dx$$

$$\begin{aligned} u &= \ln x & v &= -\frac{1}{x} \\ du &= \frac{1}{x} dx & dv &= \frac{1}{x^2} dx \\ -\frac{\ln x}{x} \Big|_1^e + \int_1^e \frac{1}{x^2} dx & \\ -\frac{\ln x}{x} \Big|_1^e + \left[-\frac{1}{x} \right]_1^e & \\ \left[-\frac{\ln e}{e} - \frac{1}{e} \right] - \left[0 - 1 \right] & \\ \boxed{-\frac{2}{e} + 1} & \end{aligned}$$

$$8) \int_0^3 f'(x) g(x) dx = 6$$

$$\begin{aligned} \int_0^3 f(x) g'(x) dx & \\ u &= f(x) & v &= g'(x) \\ du &= f'(x) dx & dv &= g'(x) dx \\ f(x)g(x) \Big|_0^3 - \int_0^3 f'(x)g(x) dx & \\ [f(3)g(3) - f(0)g(0)] - 6 & \\ (5)(3) - (1)(-4) - 6 & \\ \boxed{13} & \end{aligned}$$

$$9) \int_0^3 x f''(x) dx$$

$$\begin{aligned} u &= x & v &= f'(x) \\ du &= 1 & dv &= f''(x) dx \\ xf'(x) \Big|_0^3 - \int_0^3 f'(x) dx & \\ [xf'(x) - f(x)] \Big|_0^3 & \\ [3f'(3) - f(3)] - [0 - f(0)] & \\ (3(-1) - 5) - (-2) & \\ \boxed{18} & \end{aligned}$$

