AP Calculus BC

Integration by Parts 2

1) $\int x^2 \cos x\,dx = h(x) - \int 2x \sin x\,dx$

$u = x^2 \quad v = \sin x$

$du = 2x\,dx \quad dv = \cos x\,dx$

$\boxed{h(x) = x^2 \sin x}$

2) $\int x \sin(5x)\,dx$

D	I
x	$\sin(5x)$
-1	$-\frac{1}{5}\cos(5x)$
0	$-\frac{1}{25}\sin(5x)$

$\boxed{-\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C}$

3) $\int x \csc^2 x\,dx$

D	I	
x	$\csc^2 x$	
-1	$-\cot x$	$\int \cot x\,dx$
0	$-\ln\lvert\sin x\rvert$	$\ln\lvert\sin x\rvert$

$\boxed{-x\cot x + \ln\lvert\sin x\rvert + C}$

4) $\frac{dy}{dx} = x\sec^2 x \quad y = 1,\ x = 0$

D	I
x	$\sec^2 x$
-1	$\tan x$
0	$-\ln\lvert\cos x\rvert$

$y = x\tan x + \ln\lvert\cos x\rvert + C$

$1 = 0 + \ln\lvert\cos 0\rvert + C$

$1 = C$

$\boxed{y = x\tan x + \ln\lvert\cos x\rvert + 1}$

5) $\int_0^{\pi} t \sin 3t\,dt$

D	I
t	$\sin 3t$
-1	$-\frac{1}{3}\cos 3t$
0	$-\frac{1}{9}\sin 3t$

$\left[-\frac{1}{3}t\cos 3t + \frac{1}{9}\sin 3t\right]\Big|_0^{\pi}$

$\left[-\frac{1}{3}(\pi)(-1) + 0\right] - \left[0 + 0\right]$

$\boxed{\frac{\pi}{3}}$

6) $\int_0^{1} (x^2+1)e^{-x}\,dx$

D	I
x^2+1	e^{-x}
$-2x$	$-e^{-x}$
$+2$	e^{-x}
0	$-e^{-x}$

$\left[-e^{-x}(x^2+1) - e^{-x}(2x) - e^{-x}(2)\right]\Big|_0^{1}$

$\left[-\frac{2}{e} - \frac{2}{e} - \frac{2}{e}\right] - \left[-1 - 2\right]$

$\boxed{-\frac{6}{e} + 3}$

7) $\int_1^e \frac{\ln x}{x^2}\,dx$

$u = \ln x \qquad v = -\frac{1}{x}$

$du = \frac{1}{x}\,dx \qquad dv = \frac{1}{x^2}\,dx$

$-\frac{\ln x}{x}\Big|_1^e + \int_1^e \frac{1}{x^2}\,dx$

$-\frac{\ln x}{x}\Big|_1^e + \left[-\frac{1}{x}\right]\Big|_1^e$

$\left[-\frac{\ln e}{e} - \frac{1}{e}\right] - [0 - 1]$

$\boxed{-\frac{2}{e} + 1}$

8) $\int_0^3 f'(x)\,g(x)\,dx = 6$

$\int_0^3 f(x)\,g'(x)\,dx$

$u = f(x) \qquad v = g(x)$

$du = f'(x)dx \qquad dv = g'(x)dx$

$f(x)g(x)\Big|_0^3 - \int_0^3 f'(x)\,g(x)\,dx$

$[f(3)g(3) - f(0)g(0)] - 6$

$(5)(3) - (1)(-4) - 6$

$\boxed{13}$

9) $\int_0^3 x\,f''(x)\,dx$

$u = x \qquad v = f'(x)$

$du = 1 \qquad dv = f''(x)dx$

$x\,f'(x)\Big|_0^3 - \int_0^3 f'(x)\,dx$

$[x\,f'(x) - f(x)]\Big|_0^3$

$[3f'(3) - f(3)] - [0 - f(0)]$

$(3(7) - 5) - (-2)$

$\boxed{18}$